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# Preserving Change Information in Multi-temporal Choropleth Maps Through an Extended Data Classification Method

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## ABSTRACT

Diverse user requirements has led to an increasing availability of multi-temporal data, the analysis of which often requires visualization, e.g. in multi-temporal choropleth maps. However, if using standard data classification methods for the creation of these maps, problems arise: significant changes can be lost by data classification (change loss) or non-significant changes can be emphasized (change exaggeration). In this paper, an extended method for data classification is presented, which can reduce these effects as far as possible. In the first step, class differences are set for important or necessary changes. The actual data classification considers these class differences in the context of a sweep line algorithm, whose optimal solution is determined with the help of a measure called Preservation of Change Classes (POCC). By assigning weights during computation of this measure, different tasks or change analyses (e.g. emphasize only highly significant changes) can be processed.

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## KEYWORDS

Multi-temporal; choropleth map; data classification; change detection

## Key Policy Highlights

Due to increasing demands and data availability, more and more multi-temporal choropleth maps are generated. Standard data classification methods, however, do not consider the preservation of changes or the unwanted exaggeration of changes in these maps. A novel, extended data classification method is presented that – based on a definition of ‘important’ and ‘unimportant’ changes – enables their preservation to the highest possible degree. To control and to evaluate the process, the measure Preservation of Changes Classes (POCC) is introduced.

## Introduction

### Relevance

In many application areas such as weather, environment, traffic, health or disaster management, the demand for multi-temporal geoinformation with ever higher temporal resolutions is increasing. In response, more and more multi-temporal geospatial data (also time series data) is also being generated, e.g. by various sensors and sensor networks, remote sensing systems, or even user-generated data collection (Bill *et al.*, 2022). With this demand and availability, the need for powerful analytics is also increasing. For this purpose, the currently heavily researched artificial intelligence or machine/deep learning methods appear to be a promising option. However, there is also consensus that it is meaningful to integrate human knowledge not only for the final communication of results, but also for effective analysis or exploration (so-called human-in-the-loop; Meng, 2020). Consequently, there is also an increased need for effective and efficient visualization of multi-temporal data for presentation or exploration purposes.

In this context, choropleth maps organized by time are frequently encountered in the media and science. As an example, Mooney and Juhász (2020) found that choropleth maps were the pre-dominant type for representing the occurrence of COVID-19 (although often unjustified due to the use of unsuitable, political enumeration units; Rezk and Hendaway, 2023). Multi-temporal choropleths are usually presented as cartographic animations or map series (Slocum *et al.*, 2009). Although animations have clear disadvantages in terms of perceptibility, they are a powerful tool for communicating at least trends in spatio-temporal data (Rensink, 2002). The static representation as map series (in particular, small

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multiples) offers more flexible and deeper insights in the data, but at the cost of the impression of a real temporal sequence.

Multi-temporal choropleths represent either the attribute values for each epoch, or the difference or change between two epochs. While in the first variant the changes are not explicitly visualized and have to be mentally grasped by the user, in the second variant the original attribute values are lost. The addition of symbols to attribute value maps, which describe the change e.g. by bars, provides a remedy. At the same time, however, this combined design generates increased map complexity, which can lead to perception problems, especially with cartographic animations.

### Problem Setting

When evaluating the usability of multi-temporal choropleth maps, quite often the problem of change blindness (Harrower, 2007; Fish *et al.*, 2011) is addressed. It describes the effect that not all change information can be captured when sequentially viewing the maps, which is due to the limited human working memory. This occurs both with static map series and, in particular, with cartographic animations. The effect depends among other things on the number of changes, the variance of changes (value increase versus decrease, changes in small intervals, and so on), the number of epochs, the display time for an epoch and last but not least on the data classification method (i.e. the number, width and placement of classes).

As there is no standard or ‘optimal’ method for the data classification of any mono-temporal scene, this also applies – and even more so – to multi-temporal scenes due to different data distributions and change tasks. In fact, especially in media maps, the equidistant classification is the most common variant because it corresponds to the expectations of laypersons (Mooney and Juhász, 2020).

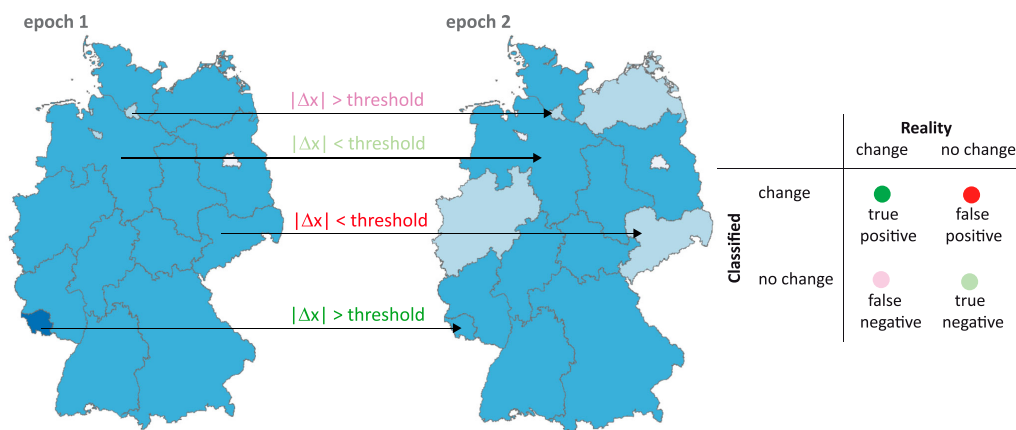
Much more elementary, however, is the problem that significant changes can be lost by data classification (referred to as change loss in the following) or non-significant changes can be unintentionally emphasized (change exaggeration). Avoiding these effects, which often work against each other (Figure 1), is of course a prerequisite for an effective detection of changes. Surprisingly, this topic has hardly or not at all been dealt with in the literature so far.

The standard methods for data classification (such as equidistant or quantiles) determine the class boundaries based on the histogram along the number line, but change information as such is explicitly not considered and thus possibly lost.

### Goal

The overall aim of this paper is to develop a classification method for multi-temporal data that reduces the described effects of change loss and exaggeration. Such a method must be able to handle different tasks concerning the value changes – e.g. the tasks of representing only the largest (highly significant) changes or all changes above a given threshold.

Optimizing the data classification for multi-temporal choropleth mapping needs to consider a careful definition of change tasks and a-priori temporal-thematic generalization operations. The resulting change loss or exaggeration effects have to be compared to existing methods using different preservation measures.



**Figure 1.** Problem setting: Fictional, bi-temporal dataset shown in arbitrary data classification; change event is based on comparison of value difference  $\Delta x$  between epochs and for each region with pre-defined threshold – leading to four exemplary cases of confusion matrix on the right (e.g. upper case reads: although threshold is exceeded, no change is shown in visualization = false negative).

## Outline

In the following, previous work on the basic problems of multitemporal choropleth maps and on task-oriented data classification is presented (Section 2). Section 3 describes the flexible data classification method, including the necessary preservation measures. The application of this method to typical datasets is demonstrated in Section 4, varying relevant parameters such as change task, number of epochs or number of classes. A discussion of transferability and parameter settings is given in Section 5. Finally, Section 6 summarizes the results and gives an outlook on future work.

## Previous Work

### Multi-temporal Choropleth Maps

Multi-temporal choropleth maps can basically appear in two forms – as cartographic animations or as static map series (in particular, small multiples). Cartographic animations gained prominence in the early 1990s, brought about by the increased power of computers, graphics cards, and so on (Campbell and Egbert, 1990; Peterson, 1995). Di Biase *et al.* (1992) provided a comprehensive overview on the design of such animations, while Blok (2000) described an assignment of appropriate dynamic visualization parameters to selected tasks. Cybulski (2022) stated that from a cartographic and psychological point of view there are still considerable deficits in describing the recognition of temporal trends of spatial patterns in animations. A comparison between animated maps with static map series is given, for example, in Griffin *et al.* (2006).

The effect of change blindness, i.e. the incomplete recognition of multi-temporal information due to the limited capacity of the human working memory, usually refers to the representation in cartographic animations. As an example, Rensink (2002) holds that a maximum of four to five elements can be perceived simultaneously. In a weakened form, however, the effect can also be transferred to static map series. There are numerous contributions in the literature that deal with this effect and the possible design causes for it (e.g. Harrower, 2007). The basic problem is that the number of visual stimuli is too large (i.e. the cognitive load is high). This can be caused in multi-temporal choropleth maps especially by large numbers of enumeration units, class values (and colours, respectively), epochs or change events as well as a high animation speed. In particular, simultaneous and opposite changes at different locations in the map are very difficult to capture. Another unwanted effect are false alarms: Users think they have detected colour changes, although they do not occur at all. This is often true for regions surrounded by regions with large changes (Cybulski and Krassanakis, 2021).

A number of graphical or interactive design measures are recommended to avoid change blindness. In particular, these include the integration of interactive elements for fast-forwarding and rewinding (e.g. through sliders; Harrower and Fabrikant, 2008). To avoid additional eye movements within a single time frame, alternative temporal legends are recommended – such as centring these legends or using speech or sound (Kraak *et al.*, 1997; Muehlenhaus, 2013). Furthermore, the transition between frames can be varied: in so-called tweening, interpolation or smoothing takes place between map frames of an animation, thereby extending the respective display duration. Whereas Fish *et al.* (2011) found that this measure actually improved readability, Simons (2000) detected underestimations of large changes due to tweening and recommended abrupt changes for this purpose.

As an alternative or addition to the aforementioned design measures, there is also the possibility of reducing the problem of change blindness by a priori data transformation or data generalization – this can refer to both spatial and temporal dimensions (Panopoulos *et al.*, 2003). Typical examples for the latter dimension are temporal selection (of specific epochs), aggregation (e.g. building monthly average of daily values) or smoothing (e.g. of daily data with 5-day windows by averaging values of four days before and the current day). Harrower (2001) recommended temporal smoothing; however, McCabe (2009) and Traun *et al.* (2021) found no improved perception through this approach. Traun *et al.* (2021) successfully performed a generalization that excluded local outliers. Traun and Mayrhofer (2018) reduced visual complexity in advance by a generalization based on spatiotemporal autocorrelation that eliminates ‘visual noise’, thus preserving both large-scale patterns and local variations. In general, data generalization also causes loss of information or misinterpretation of data; for example, Becontyé *et al.* (2022) investigated the effect of different levels of aggregation in choropleth maps. Any data classification has to be treated separately for individual applications – an example of processing COVID-19 datasets is given by Halpern *et al.* (2021).

### Task-oriented Data Classification

The topic of data classification for cartographic purposes is treated extensively in the literature – here one can refer to the overview contributions of Cromley and Cromley (1996) or Coulsen (1987). The clear focus is on

data-driven methods for static displays. It is therefore not surprising that only methods of this kind are implemented in common GIS or mapping software. In addition, interactive tools have been developed to find the ‘optimal’ configuration for a given application; e.g. the use of linked views between data histogram and choropleth map (Andrienko *et al.*, 2001).

Focusing on the spatial component only, the task to preserve spatial patterns in mono-temporal maps can be seen as a somehow similar problem to the one tackled in this paper. Common data-driven methods do not guarantee the preservation, so that in practice the typical case is a manual subjective selection from several created variants. An overview of task-oriented approaches to address this problem is given by Armstrong *et al.* (2003). Often, the goal is to simplify the displayed patterns so that the map user can more quickly grasp the broad trends in the display without being disturbed by a detailed and ‘spotty’ impression (Cromley, 1996; Andrienko *et al.*, 2001; Slocum *et al.*, 2009). However, this approach does not guarantee that any significant pattern that may be present is preserved. Hence, Chang and Schiewe (2018) developed methods to preserve specific patterns such as the detection of differences in values between polygons, hot and cold spots, global or local extreme values, or cluster regions.

A specific treatment of multi-temporal data classification is hardly done in the literature. One of the few exceptions is the work of Monmonier (1994), which aimed at minimizing small or unwanted class changes (but did not lead to satisfying results either). Harrower (2003) recommended a strong aggregation into two or three classes, which is, however, an overly simplified option for many applications.

Brewer and Pickle (2002) suggested that – applying different existing classification methods – quantiles showed best results for map comparison purposes. However, they tested only a three-class variant and found that many undesired class breaks occurred between same values so that a a-posteriori adjustment was applied. Different change tasks were not taken into consideration; for example, the quantile method will be optimal to represent only very few, largest changes. The authors also experienced that matched legends across all epochs of the time series increased the map comparison accuracy significantly (by 28 percent).

Summarizing these findings, one can state that the impact of change loss and change exaggeration in the course of data classification is rarely, if ever, discussed in the literature. On the other hand, however, this aspect should represent a fundamental basis on which further improvements for the perceptibility of changes in choropleth maps can then be developed. Simply speaking, if the aim is to detect changes, they should remain visible after data classification.

There are, of course, empirical findings for selecting appropriate classification methods for mono-temporal scenes that can consider, for example, known data distributions (e.g. the pre-knowledge that equidistant distributions are not suitable for strongly skewed distributions such as the population density in Germany or in other countries). On the other hand, a dataset for multi-temporal scenes is now extended even further by the fact that very many and different changes occur – and with that a proper assignment of a known classification method is usually not possible.

## Method

In order to find an appropriate, task-oriented visualization format for multi-temporal choropleth maps, firstly the desired change task should be defined as clear as possible. The next section describes measures for the impact of data classification on change preservation. These measures are then used to control a new data classification algorithm.

### Change Tasks

The definition of the change task includes the selection of epochs, i.e. the first and last epoch as well as the temporal resolution. The definition of temporal resolution can be based on a predefined temporal lag (e.g. considering only dates with a lag of two months) or on thematic relevance (e.g. considering only the summer months for vegetation applications).

Depending on the application, but also on the information content of the data, a user is interested in very different types of changes. If we first look at the values  $x_{t1}$  and  $x_{t2}$  for two lags at a certain enumeration unit, we can differentiate local changes such as:

- simple value changes (i.e.  $|\Delta x| = |x_{t1} - x_{t2}| > 0$  between lags),
- significant value change (i.e.  $|\Delta x| >$  given threshold, based on statistical analysis, and so on),
- change in tendency (e.g. increase, constant, decrease),
- epoch of first/last (significant) change (appearance, decay),
- epochs of all (significant) changes,
- (maximum, minimum) duration of all (significant) changes.

Of course, combinations of these types are also possible. If not only one enumeration unit at a time is considered, changes in patterns may also be of interest, such as growth or shrinkage of clusters or hot/cold spots (focal or global changes). In the remainder of this paper, however, we will focus on local changes only.

Based on the desired type of local changes, appropriate mathematical, statistical or application-oriented measures of changes have to be introduced. Possible measures are:

- (Absolute) difference between values of two different epochs of time ( $\Delta x = x_{t1} - x_{t2}$ ; or  $|\Delta x|$ ),
- (absolute) quotient of values of two different epochs of time ( $x_{t1} / x_{t2}$ ; or  $|x_{t1} / x_{t2}|$ ),
- (absolute) difference between values and trend for one location at given epoch (the trend might be derived from temporal smoothing of the time series).

## Preservation of Changes (POCC)

### Change of Values and Classes

The general task of mono-temporal data classification is to assign class values  $c$  ( $c = 1, \dots, k$ ;  $c \in \mathbf{N}$ ) to given attribute values  $x$  ( $x = x_{\text{MIN}}, \dots, x_{\text{MAX}}$ ;  $x \in \mathbf{R}$ ). The class assignment can be described in a general manner as follows:

$$c = \begin{cases} 1 & \text{for } x \leq x_1 \\ 2 & \text{for } x > x_1 \text{ AND } x \leq x_2 \\ (\dots) & \end{cases} \quad (1)$$

Hence, the task is to determine the class limits  $x_i$  ( $i = 1, \dots, k$ ). As an example, for the equidistant method the following condition has to be fulfilled:

$$x_2 - x_1 = x_3 - x_2 = \dots \quad (2)$$

In the case of multi-temporal datasets, i.e. with data for given epochs  $t$  ( $t = 1, \dots, n$ ), the formulation of the task has to be extended: For given attribute values at a enumeration unit at time  $t$  ( $x_t$ ;  $x \in \mathbf{R}$ ) respective class values  $c$  ( $c \in \mathbf{N}$ ) are required. It is assumed that there is one common classification scheme for all epochs of the time series in order to ensure comparability (Brewer and Pickle, 2002).

With that one also obtains class (or rank) differences  $\Delta c$  for given value differences  $\Delta x$  between two epochs. The overall aim is to define a set of class breaks that is able to preserve as many significant value differences (for example, to avoid change loss for large differences), and/or to avoid class breaks between very small changes (i.e. to avoid change exaggeration).

When transforming given value differences  $\Delta x$  into required class differences  $\Delta c_{\text{required}}$  several approaches are conceivable depending on the change task. The evaluation of the preservation can be done via different measures.

### Selected Variants

When transforming given value differences  $\Delta x$  into required class differences  $\Delta c_{\text{required}}$ , several approaches are conceivable depending on a certain change task. In the following, three important variants are presented as examples, which describe an upper threshold value  $\Delta x_{\text{UPPER}}$  for the class with the 'most important' changes. The classification of the remaining change values is done in different ways (Figure 2). The definition of the necessary threshold values is for example based on statistical parameters (e.g. using  $2\sigma$ -,  $1\sigma$ -, etc. values, or any percentiles) or by taking application-dependent definitions of 'important changes' into consideration.

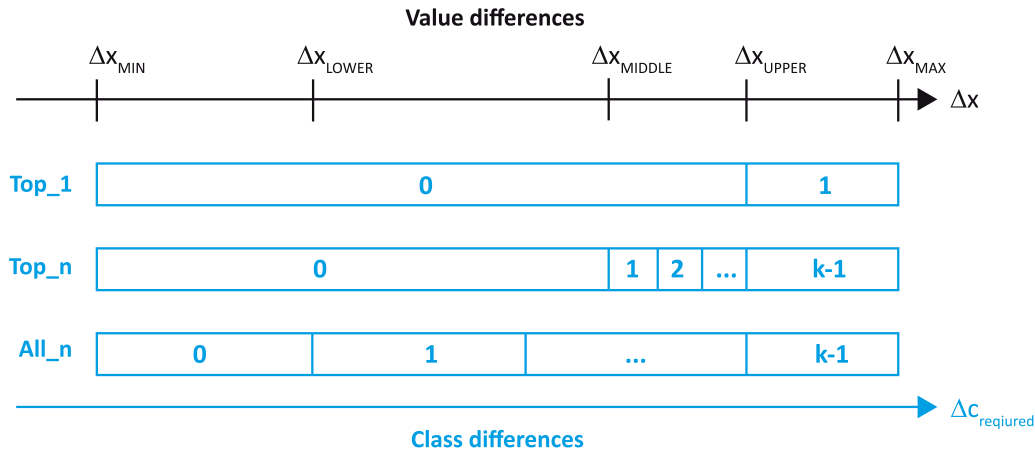
With the 'Top\_1' variant, all changes  $\Delta x$  above a threshold value  $\Delta x_{\text{UPPER}}$  receive a class difference  $\Delta c_{\text{required}} = 1$ :

$$\Delta c_{\text{required}} = 1 \text{ for } |\Delta x| > \Delta x_{\text{UPPER}} \quad (3)$$

$\Delta x_{\text{UPPER}}$  can be defined, for example, using a quantile of the entire dataset (e.g. the 90% quantile). If  $\Delta x_{\text{UPPER}} = \Delta x_{\text{MAX}}$  were set, only this maximum difference would be included in the top class change class.

With the variant 'Top\_n', all changes  $\Delta x$  above a threshold value  $\Delta x_{\text{MIDDLE}}$  are transferred into several, equally spaced class jumps. Again, the top class starts at the threshold value  $\Delta x_{\text{UPPER}}$ . If the classification is calculated for  $k$  classes, the required class difference is given by:

$$\Delta c_{\text{required}} = \begin{cases} \text{INT} \left( \frac{k-2}{\Delta x_{\text{UPPER}} - \Delta x_{\text{MIDDLE}}} (\Delta x - \Delta x_{\text{MIDDLE}}) + 1 \right) & \text{for } \Delta x > \Delta x_{\text{MIDDLE}} \\ 0 & \text{else} \end{cases} \quad (4)$$



**Figure 2.** Variants of the assignment of class differences  $\Delta c$  depending on value differences  $\Delta x$ .  $\Delta x_{MIN}$  and  $\Delta x_{MAX}$  are the extreme value changes within dataset; the other thresholds are determined based on variants as described in the text.

Both  $\Delta x_{MIDDLE}$  and  $\Delta x_{UPPER}$  can be set, for example, by using quantiles of the entire dataset (e.g. the 70% and 90% quantiles).

With the ‘**All\_n**’ method, all changes  $\Delta x$  above a threshold value  $\Delta x_{LOWER}$  are transformed into several, equally spaced class differences. The top class starts at the threshold value  $\Delta x_{UPPER}$ . If the classification is calculated for  $k$  classes, the required class difference as a special case of ‘**Top-n**’ (with  $\Delta x_{LOWER} = 0$ ) results from

$$\Delta c_{required} = INT \left( \frac{k-1}{\Delta x_{UPPER}} \Delta x \right) \quad (5)$$

### Evaluation of Preservation

After applying any data classification method, the requested class differences  $\Delta c_{required}$  can be compared with the differences  $\Delta c_{achieved}$  that have actually been obtained. For that purpose, simply the absolute or relative number of preserved change classes could be counted. A more meaningful measure is based on the metrics as introduced by Goldsberry and Battersby (2009). They counted either the number of enumeration units whose class changed (somehow) between two epochs (Basic Magnitude of Change; BMOC), or the total number of cardinal or ordinal classes that changed between two epochs, also taking into account the magnitude of the possible class difference (Magnitude Of Rank Change; MORC). Figure 3 demonstrates the respective weighting factors for the BMOC- and MORC-measures (for an example with three classes).

In contrast to Goldsberry and Battersby (2009), in this contribution we do not consider the change of values  $x$ , but the preservation of change classes (POCC)  $\Delta c$ . To do this, the number of required class differences for a given value interval ( $\Delta c_{required}$ ) is compared to the actual class interval achieved by data classification ( $\Delta c_{achieved}$ ; Figure 3). A normalization is realized by dividing this difference by  $\Delta c_{required}$ , resulting in the following *POCC* measure:

$$POCC = 1 - \frac{\sum_i w_i \cdot |\Delta c_{required,i} - \Delta c_{achieved,i}|}{\sum_i w_i \cdot \Delta c_{required,i}} \quad (6)$$

<b>BMOC</b>		<b>Epoch 2</b>			<b>MORC</b>		<b>Epoch 2</b>		
		Class 1	Class 2	Class 3			Class 1	Class 2	Class 3
<b>Epoch 1</b>	Class 1	0	1	1	<b>Epoch 1</b>	Class 1	0	1	2
	Class 2	1	0	1		Class 2	1	0	1
	Class 3	0	0	1		Class 3	0	1	2

<b>POCC</b>		$\Delta c_{achieved}$		
		$\Delta c = 0$	$\Delta c = 1$	$\Delta c = 2$
$\Delta c_{required}$	$\Delta c = 0$	0 (or w)	1·w	2·w
	$\Delta c = 1$	1·w	0 (or w)	1·w
	$\Delta c = 2$	2·w	1·w	0 (or w)

**Figure 3.** Comparison of weights for measures BMOC and MORC (according to Goldsberry and Battersby, 2009) and for measure POCC (including weights  $w$  – see text below).

The index  $i$  runs over all pairs of values with a predefined lag (by default lag = 1, i.e. consecutive pairs). The larger POCC (with maximum 1), the better the preservation. The weights  $w$  can be used to emphasise certain class preservations. In the ‘Top\_1’ variant, the weights  $w$  are set as follows:

$$w_i = \begin{cases} 1 & \text{for } \Delta c_{\text{required}} = 1 \\ 0 & \text{else} \end{cases} \quad (7)$$

In contrast, for variants ‘Top\_n’ and ‘All\_n’ the weights are set as follows:

$$w_i = \Delta c_{\text{required}} \quad (8)$$

The differences between  $\Delta c_{\text{required}}$  and  $\Delta c_{\text{achieved}}$  in POCC consider false negatives (i.e. change loss). By introducing a weighting of the no-change classes ( $\Delta c_{\text{required}} = 0$ ), however, the false positives (i.e. change exaggeration) can also be considered – for the variant ‘Top\_1’ by setting for all differences

$$w_i = 1 \quad (9)$$

as well as for variants ‘Top\_n’ and ‘All\_n’ by:

$$w_i = \begin{cases} k - 1 & \text{for } \Delta c_{\text{required}} = 0 \\ \Delta c_{\text{required}} & \text{else} \end{cases} \quad (10)$$

Alternatively, the false positives can also be described by their relative number directly – either in comparison to the total number of all changes (FP1), or in comparison to the number of all  $\Delta c_{\text{required}} > 0$  (FP2). In general, the larger FP1 or FP2 are, the larger the proportion of unwanted false positives. If, for example,  $FP2 > 1$ , more than half of all class changes are not due to requested changes at all and thus do not allow the isolated detection of significant changes.

Grouping into classes generally leads to a loss of original values. In order to determine the degree of these losses (also in comparison to other methods), the simple measure Preservation of Original Values (POOV) is used in the following. For this purpose, the dispersion of the original values  $x$  is determined for each epoch by means of the standard deviation ( $RMSE_x$ ). This value is divided by the predefined width of equidistant of classes ( $\Delta x_{\text{equidistant}}$ ), which corresponds to the standard deviation of a corresponding equidistant grouping. This reference value is compared to the standard deviation of the class values ( $RMSE_{\text{Class}}$ ) for each epoch in the (POCC or any other) classification:

$$POOV = 1 - \frac{1}{n} \sum_{t=1}^n \left| \frac{RMSE_x}{\Delta x_{\text{equidistant}}} - RMSE_{\text{Class}} \right| \quad (11)$$

The larger POOV (with the maximum POOV = 1), the better the dispersion of the original values is preserved after classification.

### POCC Data Classification

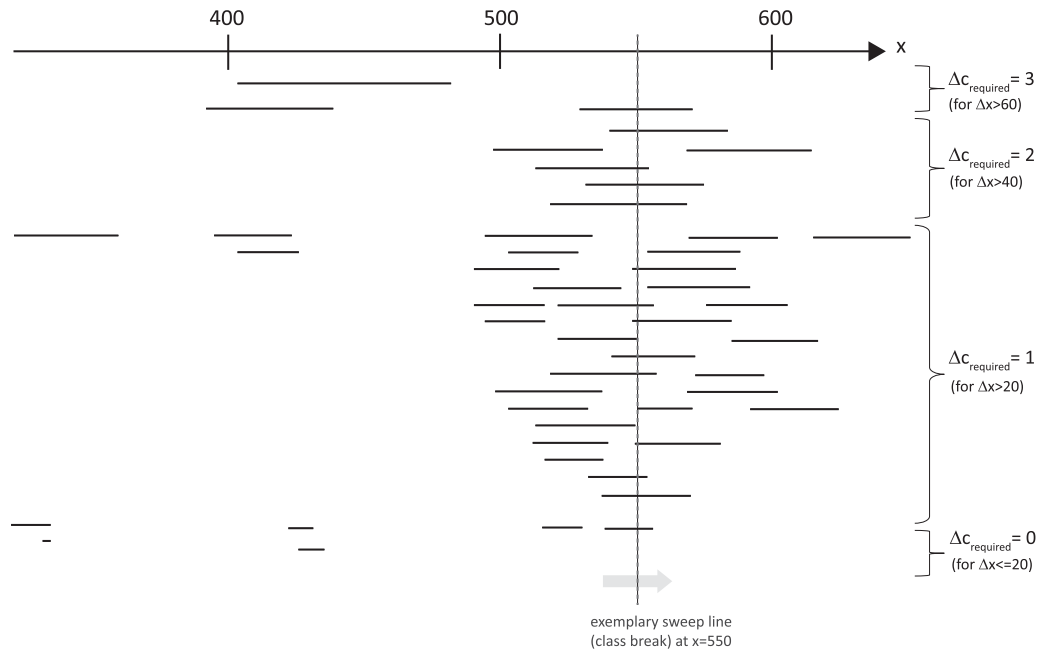
The determination of class limits for given attribute values  $x$ , which also considers the aforementioned determined required change classes  $\Delta c_{\text{required}}$ , is performed through a sweep line algorithm. The corresponding diagram (Figure 4) shows attribute values  $x$  on the right axis, while all attribute differences  $\Delta x$  between two lags are plotted as intervals below. These differences are already grouped according to the respective required class difference  $\Delta c_{\text{required}}$ .

The sweep line is moved from left to right over the intervals. For this purpose, discrete steps must be defined for the sweep line based on the properties of the dataset and the resulting computation time. The strictest option is to choose this resolution smaller than the smallest difference between the values  $x$  occurring in the dataset.

An intersection of a sweep line with such an interval represents a possible class boundary between the lower and upper breaks of the interval. For each sweep line, the corresponding number of intersections with value intervals  $|\Delta x|$  can be counted. At the end, for each interval the total number of intersections (i.e. the class difference  $\Delta c_{\text{achieved}}$ ) is counted and compared to the required value  $\Delta c_{\text{required}}$  using the POCC measure (section 3.2.3).

Given an a priori fixed number of change classes, the optimal solution is obtained using a brute force approach that considers all possible combinations of sweep lines (i.e. all class boundaries) and the respective preservation measures POCC. The best combination is then selected based on the maximum POCC value. In the following, this novel method will also be referred as *POCC data classification* method.





**Figure 4.** Sweep line diagram for given dataset: line segments below number line (representing values  $x$ ) show placement and length of all value intervals  $|\Delta x|$ . This arbitrary example for variant ‘All\_n’ requires class differences  $\Delta c$  (shown on right hand side) for  $\Delta x_{UPPER} = 60$  and  $\Delta x_{LOWER} = 20$ .

## Example

### Dataset

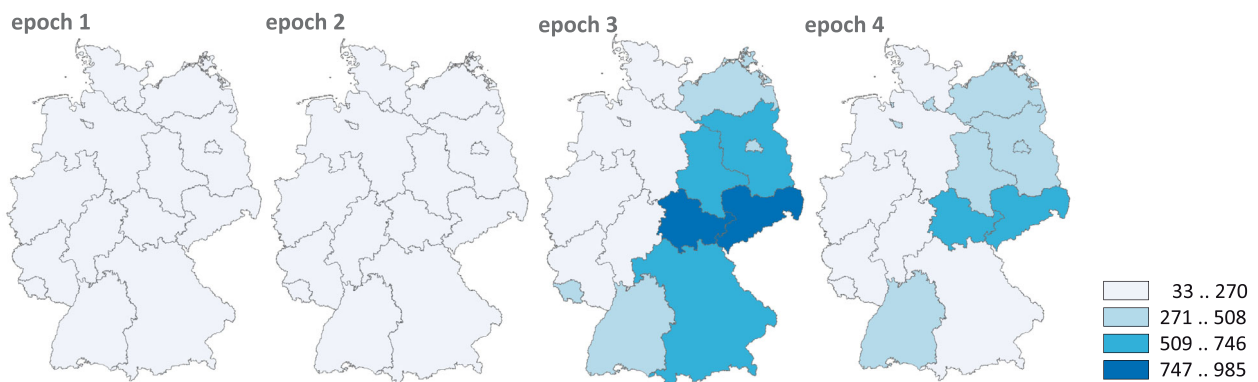
The Robert Koch Institute (Berlin, Germany) continuously publishes various spatiotemporal data on COVID-19 cases on its website. In the following, the dataset for daily calculated 7-day incidences for each of the 16 federal states in Germany is used (between 11th September 2021 and 17th April 2023).

To reduce this dataset, temporal aggregation was performed – a maximum of 19 epochs were thus generated by monthly averaging. For comparison purposes, also subsets with four epochs (Figure 5) and six epochs were created.

### POCC Classification

The POCC data classification described above was performed for all combination of the following parameter variations:

- POCC-Variants: ‘Top\_1’, ‘Top\_n’ and ‘All\_n’;
- Number of classes: 3, 4 and 5. As stated above, Harrower (2003) recommended a strong aggregation into two or three classes; however, this overly simplified option is extended to more typical values for cartographic animations;
- Number of epochs: 4, 6 and 19.



**Figure 5.** Example dataset on monthly COVID-19 incidences – four epochs, equidistant classification with four classes (data source: [https://www.rki.de/DE/Content/InfAZ/N/Neuartiges\\_Coronavirus/Daten/Inzidenz-Tabellen.html](https://www.rki.de/DE/Content/InfAZ/N/Neuartiges_Coronavirus/Daten/Inzidenz-Tabellen.html)).

The absolute difference  $|\Delta x|$  was used as a measure of the change between two epochs. To determine the threshold values,  $\Delta x_{\text{UPPER}}$  was equated with the 90% quantile and  $\Delta x_{\text{MIDDLE}}$  with the 70% quantile of the dataset.

The evaluation is carried out using the measures listed above (POCC, POOV, FP1, FP2). For comparison purposes, the corresponding measures for the equidistant classification method were also determined. Tables 1–4 summarise the results, sorted by the variants ‘Top\_1’, ‘Top\_n’ and ‘All\_n’ (the latter with the different weighting methods). Parallel to these tables, Figures 6–9 show an example of the resulting choropleth map series for one case (four epochs, four classes). In the figures, the assignment of true/false and positive/negative is marked, independent of the magnitude of the required class difference.

## Discussion

The following discussion evaluates the POCC data classification based on the introduced measures – namely, preservation of class changes (POCC), preservation of dispersion of input data (POOV) and the relative number of false positives (FP1 and FP2).

**Table 1.** Results of variant ‘Top\_1’ ( $w_i = \Delta c_{\text{required}}$ ).

Epochs	Classes	Class breaks POCC	POCC		POOV		FP1		FP2	
			POCC	equi	POCC	equi	POCC	equi	POCC	equi
4	3	130/607	1.00	.60	.76	.87	.29	.21	2.80	2.00
	4	54/130/608	1.00	–.20	.60	.92	.38	.29	3.60	2.80
	5	54/68/130/608	1.00	–.40	.40	.94	.33	.33	3.20	3.20
6	3	535/1557	1.00	.50	.89	.95	.24	.31	2.38	3.13
	4	54/535/1557	1.00	–.25	.76	.91	.31	.35	3.13	3.50
	5	54/68/535/1557	1.00	–.88	.62	.91	.39	.38	3.88	3.75
19	3	514/1557	1.00	.50	.23	.39	.32	.20	15.33	9.67
	4	23/514/1557	1.00	.00	.08	.16	.32	.35	15.33	16.67

**Table 2.** Results of variant ‘Top\_n’ ( $w_i = \Delta c_{\text{required}}$ ).

Epochs	Classes	Class breaks POCC	POCC		POOV		FP1		FP2	
			POCC	equi	POCC	equi	POCC	equi	POCC	equi
4	3	262/565	.82	.61	.87	.87	.06	.06	.20	.20
	4	262/536/565	.71	.61	.78	.92	.02	.13	.07	.40
	5	251/339/536/565	.70	.50	.72	.94	.10	.17	.33	.53
6	3	641/946	.79	.70	.87	.95	.25	.11	.83	.38
	4	457/641/946	.68	.58	.82	.91	.34	.15	1.12	.50
	5	641/714/874/936	.62	.48	.72	.84	.22	.15	.75	.50
19	3	614/1285	.84	.79	.36	.46	.17	.09	1.20	.68
	4	726/1040/1748	.81	.73	.39	.24	.11	.23	.82	1.65

**Table 3.** Results of variant ‘All\_n’ (no weighting for no-class changes).

Epochs	Classes	Class breaks POCC	POCC		POOV		FP1		FP2	
			POCC	equi	POOV	equi	POCC	equi	POCC	equi
4	3	262/565	.84	.62	.87	.87	.06	.02	.25	.05
	4	262/339/565	.73	.63	.52	.92	.05	.04	.08	.09
	5	262/268/403/565	.72	.49	.60	.94	.02	.00	.04	.00
6	3	427/946	.87	.77	.89	.96	.06	.04	.16	.08
	4	427/641/946	.78	.65	.84	.91	.04	.01	.08	.02
	5	427/641/874/1044	.76	.63	.88	.85	.04	.00	.07	.00
19	3	614/1285	.84	.80	.35	.50	.18	.03	1.38	.11
	4	514/972/1404	.85	.82	.26	.35	.14	.06	.52	.18

**Table 4.** Results of variant ‘All\_n’ (with weighting for no-class changes).

Epochs	Classes	Class breaks POCC	POCC		POOV		FP1		FP2	
			POCC	equi	POCC	equi	POCC	equi	POCC	equi
4	3	253/457	.69	.50	.82	.87	.08	.02	.33	.05
	4	262/403/634	.67	.54	.57	.92	.00	.04	.00	.09
	5	253/262/403/536	.68	.45	.60	.94	.04	.00	.09	.00
6	3	641/1044	.73	.63	.89	.96	.04	.04	.10	.08
	4	457/641/946	.74	.63	.82	.91	.05	.01	.11	.02
	5	457/641/874/1044	.74	.60	.86	.85	.04	.00	.07	.00
19	3	1040/1670	.29	–.25	.55	.50	.04	.03	.38	.11
	4	665/1044/1404	.66	.29	.40	.35	.05	.06	.16	.18

With regard to the preservation of desired class changes, the expected ‘superiority’ of the POCC over the equidistant classification becomes apparent for all variants. This is logical insofar as the classification is controlled according to the POCC criterion. Nevertheless, it is striking that the equidistant method never reaches this optimal value of preservation.

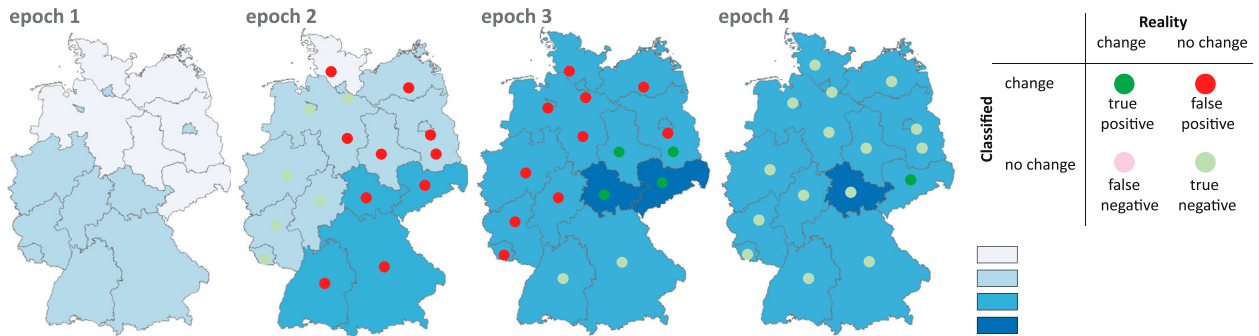


Figure 6. POCC classification for variant ‘Top\_1’ (four epochs, four classes), point symbols describe correctness of change relative to previous epoch.

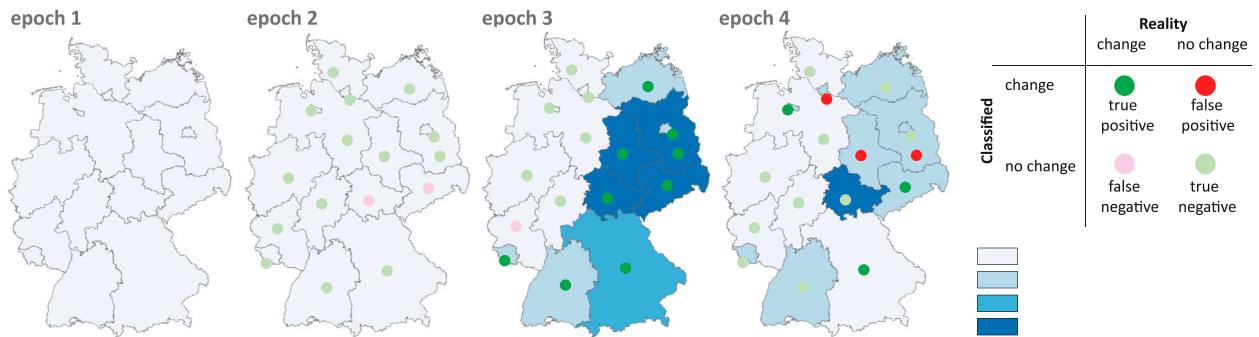


Figure 7. POCC classification for variant ‘Top\_n’.

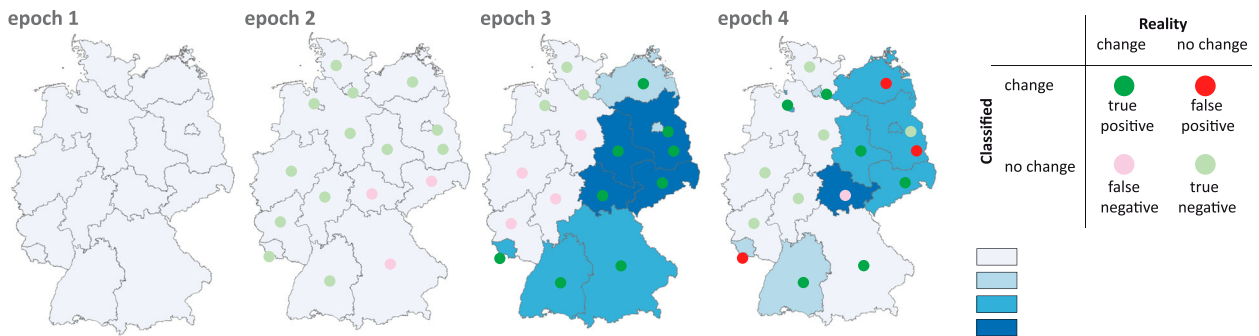


Figure 8. POCC classification for variant ‘All-n’ (no weighting for no-class changes).

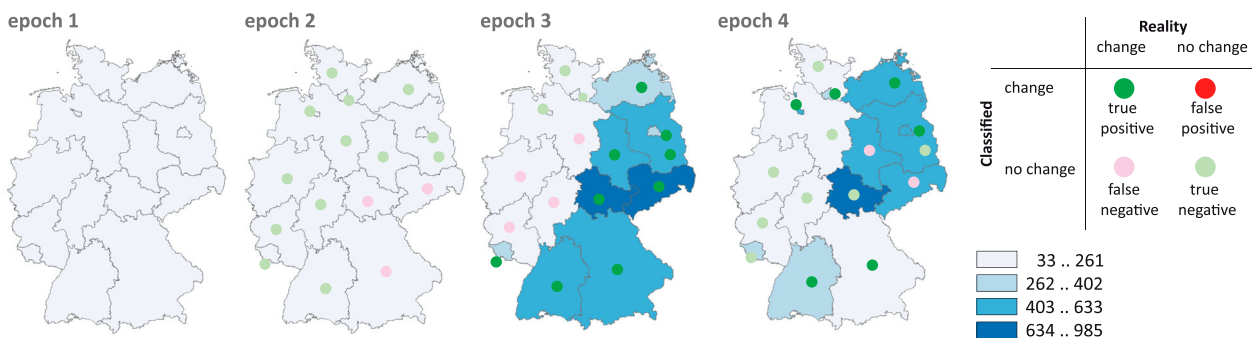


Figure 9. POCC classification for variant ‘All-n’ (with weighting for no-class changes).

If one looks at the behaviour of the POCC value as a function of the number of classes, there is no clear tendency. For the variants ‘Top\_n’ and ‘All\_n’ (Tables 2 and 3), at best a slight decrease can be observed as the number of classes increases. There is also no clear dependence on the number of epochs. There are hardly any differences between the variants ‘Top\_n’ and ‘All\_n’ (the latter without weighting of the no-changes). However, there are significantly worse values for ‘All\_n’ with weighting of the no-changes (Table 4), since a larger number of potential misclassifications are added by the no-change class and thus worsen the POCC value. In contrast, the number of cases considered is significantly lower for the variant ‘Top\_1’, which leads to 100% preservation in the given example.

When looking at the preservation of the dispersion of the input data, it becomes apparent that the POCC classification usually has quite similar, but usually slightly worse values compared to the equidistant classification. Of course, it is debatable how much the dispersion behaviour should be preserved at all, since this can compete with the highlighting of the class changes and large dispersions can rather lead to many class changes and confusion for the user. However, it can be stated that the POCC classification does not cause any significant loss of the original input data.

As already mentioned, the focus of the POCC measure is on the consideration of false negatives, i.e. those desired class changes that are not produced by a classification (i.e. change loss). On the other hand, however, false positives (FP) also significantly interfere with the interpretation of a visualization, as these unintended class changes (i.e. change exaggeration) cannot be separated from the intended changes.

The POCC measure offers a consideration through appropriate weighting – as for the variant ‘All\_n’ with the weight  $w = k-1$  for the consideration of no-changes (Table 4). As expected, the POCC numbers are reduced by this stricter measure compared to the ‘All\_n’ variant without weighting of the no-changes (Table 3).

Conversely, the measures FP1 and FP2 directly show the relative proportions of false positives. Here, the elementary weakness of the ‘Top\_1’ variant (Table 1) becomes quite clear: relatively large changes receive a class difference of 1 – but such a difference also occurs relatively frequently with smaller value differences due to the value distribution at the selected boundaries. In particular, the measure FP2 shows that the number of class differences due to relatively small differences is a multiple (by factors of 3.6 to 6.0) of the intended class differences (above the threshold  $\Delta x_{UPPER}$ ), which means that the intended detection of larger differences only is not possible.

For the other variants, there is no clear dependence of FP1 and FP2 on the number of classes or epochs. There is also no clear tendency towards the equidistant classification, which indicates random deviations depending on the real data distribution. In summary, it can be stated that the POCC classification shows neither advantages nor disadvantages with regard to the false positives compared to the equidistant classification – which is not surprising in view of the explicit non-consideration (except for the weighting of the no-changes for ‘All\_n’).

## Summary and Outlook

### Summary

In this paper, a novel extended method for data classification was presented, which explicitly considers the most complete preservation of ‘important’ and/or the avoidance of ‘unimportant’ class changes – and thus reduces the effects of change loss or change exaggeration. For this purpose, the measure POCC was introduced, which is able to control and optimize this procedure. By introducing different weights, certain change types (e.g. the highest change values) can be emphasized.

The behaviour of the measures POCC for the preservation of class changes showed the added value compared to the equidistant classification, while there were no significant deviations regarding the preservation of the original dispersion in the dataset as well as the relative number of false positives. With that, the overall aim of this paper could be fulfilled, which was the development of a classification method for multi-temporal data that reduces the effects of change loss and exaggeration and allows the consideration of different change tasks.

### Future Work

With respect to the POCC data classification algorithm some further developments are conceivable, for example:

- An essential extension should include the consideration or avoidance of the significantly disturbing false positives – beyond the previously introduced, optional weighting of the no-changes in the POCC measure.
- So far, once thresholds have been set ( $\Delta x_{UPPER}$ , and if needed also  $\Delta x_{MIDDLE}$ ,  $\Delta x_{LOWER}$ ), a proportional classification of classes in the remaining range of values has been assumed. It is conceivable that additional criteria could be used to achieve better preservation measures for non-proportional class ranges.

- In this paper, local significant value changes (described by the absolute difference  $|\Delta x|$ ) were used as the change type. Although this can certainly be seen as the most important change type, other types (such as deviation from trend) and other measures (such as quotient) must be considered, leading to adopted definitions of the POCC measure.
- The POCC measure was used as a global measure so far – depending on the application or distribution of the data, local weighting is also conceivable.
- A disadvantage of the previous implementation is the brute force approach, which sometimes leads to long calculation times (which was one of the reasons for not calculating the option ‘19 epochs/5 classes’ which takes many hours using a standard computer). Optimizations and approximations are necessary here.

From a methodological point of view, it should be noted that the experimental verification has so far only been carried out with one dataset. With the help of the different number of epochs, a certain variance of input values  $x$  as well as of changes  $\Delta x$  has already been achieved – nevertheless, further sample data are still to be examined. In this context, alternative threshold values should also be used (instead of  $\Delta x_{\text{UPPER}} = 90\%$  quantile or  $\Delta x_{\text{MIDDLE}} = 70\%$  quantile in this contribution).

Empirical studies will help to describe the influence and sensitivity of different parameter settings on the changes actually perceived by the user. In other words, it has to be examined whether the numerical progress as achieved with POCC classification is actually perceived by users. In this context, different types of multi-temporal visualizations (namely, cartographic animations and small multiples) have to be differentiated in future investigations.

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## Disclosure Statement

No potential conflict of interest was reported by the author(s).

## Data Availability Statement

Data used in the experiments were extracted from [https://www.rki.de/DE/Content/InfAZ/N/Neuartiges\\_Coronavirus/Daten/Inzidenz-Tabellen.html](https://www.rki.de/DE/Content/InfAZ/N/Neuartiges_Coronavirus/Daten/Inzidenz-Tabellen.html)

## Code Availability

The core code for POCC classification (usable for different variants) is stored at <https://github.com/luftj/pocc>

## Notes on contributor



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